Performance Probability

1. Introduction

The performance probability is a likelihood measure of a client reaching his/her current P&L. For example, if Client *A* has gained a profit of $30 after purchasing 3 binary options. What is the probability of achieving a profit equal to or above $30 if one re-purchases all of Client *A*'s options (with the same payout size and probability of winning)?

2. Model Description

Let X_k be the payout outcome of a client's k-th purchased binary option with the probability of winning p_k. X_k has two possible outcomes: W_k if client wins the contract, and L_k otherwise. Using the example above, Client *A*'s P&L can be modelled as the sum of all payout outcomes (i.e. $X_1 + X_2 + X_3$). Then, the performance probability is the sum of all possible sums of payout outcomes greater than the client's P&L multiplied with its corresponding probability of occurring. If the winning and losing payouts of the three contracts are {+$40, +$50, +$60} and {-$40, -$50, -$60} respectively with all winning probabilities at 50%, then the performance probability is:

$$\mathrm{P}(X_1 + X_2 + X_3 \geq P\&L)$$

$$= \mathrm{P}(X_1 + X_2 + X_3 \geq 150 \; or \; X_1 + X_2 + X_3 \geq 30) = 0.5^3 + 0.5^3 = 0.25$$

3. An approximation for large number of options

According to the Central Limit Theorem, a large sum of iid random variables converge asymptotically to the normal distribution. Let *'n'* be the number of contracts a client has purchased, and *'PL'* be the current P&L of the client. Since

$$Z = \frac{(X_1 + X_2 + \cdots + X_n) - E(X_1 + X_2 + \cdots + X_n)}{\sqrt{Var(X_1 + X_2 + \cdots + X_n)}} \sim N(0,1),$$

the performance probability is just:

$$\mathrm{P}(Z \geq PL) = 1 - \Phi(Z).$$

The computed $Var(X_1 + X_2 + \cdots + X_n)$ in the tool accounts for the covariance between time-overlapping options (i.e. options on the same underlying asset sharing a common time period, see *C&C_TimeOverlapBinaries.docx*).

4. Model assumptions

4.1. The model assumes no sell-back of options.
4.2. The model assumes all option payout outcomes are independent. If there are a large number of highly correlated options, the normality assumption does not hold.